provide nonhazardous testing, investigate field problems, and incorporate design changes. A totally integrated systems tape will be verified on the EMM and used at Seal Beach. The test and computer room uses actual field test equipment.

In the Coca area of Rocketdyne's Santa Susana Field Lab., the battleship test stand, for engine systems testing, and the common bulkhead test stand have been constructed. The first, constructed of heavy-walled stainless steel patterned after the flight-weight configuration and utilizing a double-walled, vacuum-jacketed outer shell on the LH_2 tank, permits 1) full-scale evaluation of the J-2 engine cluster performance, 2) evaluation of engine start and shutdown characteristics, 3) verification of stage system component functions, and 4) duration firing evaluation and product improvement testing.

System redundancy on all of the cyrogenic systems has been installed for complete safety of operation. A control center and service center are provided which simulate those outlying activities and utilize many similiar pieces of checkout equipment. All of the checkout operations will be manually controlled.

The common-bulkhead-test tank is a full-diameter specimen consisting of a forward skirt, LH_2 bulkhead, common bulkhead, and foreshortened LH_2 tank located between the LH_2 and common bulkheads. Primary objective of its test is to verify structural integrity of the bulkheads and external tank insulation through application of flight bending moments, critical internal pressures, leak checks, heat losses, and a full proof test to destruction. Use of cryogenic liquids will determine actual heat transfer during ground environmental tests along with repeated pressure conditions.

The all-systems-test vehicle (S-II-T) is a complete flight-weight stage that, in a static-firing test stand, permits combined static-firing and cryogenic testing of the stage systems. All systems testing represents the first use of complete automatic GSE during servicing, checkout, and countdown procedures. The planned testing will be accomplished at Mississippi Test Facilities (MTF). Specific test objectives include 1) proving system performance over simulated entire flight sequence within environmental limitations, 2) proving stage operating and handling procedures, 3) determining optimum propellant chilldown and topping rates, and base cooling and cold soak rates, 4) developing detanking and shutdown procedures, and 5) determining compliance with flight requirement specifications.

The dynamic test stage (S-II-D) will be the first stage delivered to the NASA Marshall Space Flight Center. This simulation article will be used to measure bending mode frequencies and mechanical interface with the S-IC and S-IV-B stages. It will incorporate dummy engine masses capable of gimballed movement to simulate inertial characteristics. Equipment or components not required for the tests will be replaced by dummy masses.

The facility-checkout stage (S-II-F) is a flight-weight article similar in appearance to the S-II except for rigidly mounted engine masses with simulated dummy systems and partially operable systems. Its primary mission is to verify compatibility between the S-II flight stage, GSE, and facilities. The S-II-F will be delivered directly to Merritt Island Launch Area (MILA) at Kennedy Space Center.

To provide final reliability and qualification checking of the S-II, a series of tests will be run at Seal Beach, MTF, and MILA facilities utilizing computer-controlled automatic checkout equipment that will provide a fully integrated checkout of stage systems, record checkout operations and test results and stores, and compare and verify stage-system operational capability. In the event of a malfunction, it will be possible to switch from automatic mode of checkout to a local control mode. Handling equipment, which includes large rings, fixtures, and transporters, has been compounded by the State of California regulations that forced us to design two separate complete transporters, one for use on California roadways and the other for direct delivery to NASA bases.

Interaction of a Weak Shock with a Combustion Region

G. M. Lehmann* and S. N. B. Murthy†
Purdue University, Lafayette, Ind.

Nomenclature

a	=	acoustic velocity $(\gamma RT)^{1/2}$, fps
c_v	=	specific heat at constant volume, ft-lb/lbm-°R
H	=	dimensionless heat release function, Eq. (23)
K	$\equiv$	$\{[2+(\gamma-1)M^2][2\gamma M^2+1-\gamma]\}^{1/2}$
M	=	Mach number of gas entering shock (relative to shock)
p	=	static pressure, lb/ft^2
T	=	static temperature, °R
t	=	time, sec
U_c	=	heat of combustion, ft-lb/lbm
u	=	gas velocity, fps
$\dot{w}$	=	mass flow rate per unit area, lbm/sec-ft^2
x	=	distance, ft
Y	=	concentration of reactant (ρ_r/ρ)
α	=	combination frequency and steric factor, sec^{-1}
γ	=	specific-heat ratio (c_p/c_v)
ϵ	=	activation energy, °R
ρ	=	density, lbm/ft^3

Subscripts

r	=	reactant flow property
s	=	steady-state flow property in undisturbed stream

Operators

$$\delta_+(\ )/\delta t = \partial(\ )/\partial t + (u+a)\,\partial(\ )/\partial x$$
$$\delta_-(\ )/\delta t = \partial(\ )/\partial t + (u-a)\,\partial(\ )/\partial x$$
$$\delta(\ )/\delta t = \partial(\ )/\partial t + u\,\partial(\ )/\partial x$$

Introduction

A PROBLEM of interest in the study of the unstable operation of a chemical propellant rocket motor is the formation and subsequent propagation of a weak shocklike pressure pulse in the combustion chamber. The mechanism involved in the formation of such a pulse in a flowing, chemically reactive gas is not adequately understood at present. If, however, the interaction of the rear of the pulse with its shocklike front can be neglected, the behavior of such a pulse may be examined by considering its propagation to be that of a weak normal shock.[1] Several investigators have employed either analytical or numerical methods for studying the propagation of a weak shock in a stationary gas containing a density gradient and in a stationary gas contained in a duct of variable area.[2–4] The analytical methods demonstrate that an approximate relation can be established between the change in the strength of the shock M and the change of the flow properties T, p, Y, etc. of the medium. The present note develops such a relation for the propagation of a normal shock through a chemically reactive flowing gas and compares the results obtained with those obtained by numerical integration of the conservation equations with appropriate boundary conditions.

Analysis

The combustion region of a rocket motor is three-dimensional, contains gradients in the flow properties, and is composed of a large number of chemical species. In the analysis,

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* Research Assistant, Jet Propulsion Center.
† Visiting Professor of Mechanical Engineering, Jet Propulsion Center.

however, it is assumed that the flow can be adequately represented by a flowing, one-dimensional, two-component, chemically reacting, ideal-gas mixture in which the "reactant" is converted to the "product" in a one-step exothermic reaction. The physical properties of the reactant and product are assumed not to differ significantly, and the effects of all of the transport processes in the flowing fluid are neglected.

The combustion region is assumed to be normal to the direction of the flow, and the chemical reaction rate is assumed to be given by an Arrhenius-type function. The equations for the conservation of reactant mass, mixture mass, momentum, and energy are

$$(\partial \rho_r/\partial t) + [\partial(\rho_r u)/\partial x] = -\alpha \rho_r e^{-\epsilon/T} \tag{1}$$

$$(\partial \rho/\partial t) + [\partial(\rho u)/\partial x] = 0 \tag{2}$$

$$(\partial u/\partial t) + (u\, \partial u/\partial x) = -(1/\rho)\, \partial p/\partial x \tag{3}$$

$$\rho c_v[(\partial T/\partial t) + (u\, \partial T/\partial x)] = \alpha U_c \rho_r e^{-\epsilon/T} - p\, \partial u/\partial x \tag{4}$$

The equation of state is

$$p = \rho R T \tag{5}$$

Equations (1–4) can be transformed into a system of total differential equations, termed the compatibility equations, which are satisfied along certain prescribed curves, termed the characteristic curves. The equations for the characteristic curves and the corresponding compatibility equations are as follows:

Positive characteristic curve $(dx/dt = u + a)$

$$\delta_+ p/\delta t + (a\rho)\, \delta_+ u/\delta t = (\gamma - 1)\, \alpha U_c \rho_r e^{-\epsilon/T} \tag{6}$$

Negative characteristic curve $(dx/dt = u - a)$

$$\delta_- p/\delta t - (a\rho)\, \delta_- u/\delta t = (\gamma - 1)\, \alpha U_c \rho_r e^{-\epsilon/T} \tag{7}$$

Path-line characteristic curve (streamline) $(dx/dt = u)$

$$\rho c_r\, \delta T/\delta t - RT\, \delta\rho/\delta t = \alpha U_c \rho_r e^{-\epsilon/T} \tag{8}$$

$$\delta Y/\delta t = -\alpha Y e^{-\epsilon/T} \tag{9}$$

An analytical solution for determining the effect of gradients in the flow upon the change in strength of a shock now will be obtained by applying a method proposed by Whitham.[2] That method is based upon the assumption that the changes in the flow properties in the unsteady flow behind a propagating normal shock, calculated along a positive characteristic, can be expressed in terms of the steady flow properties in front of the shock and the shock strength. This is done by substituting the Rankine-Hugoniot relations and their total derivatives into the positive compatibility equation [Eq. (6)], written in finite-difference form:

$$\delta_+ p + \langle a\rho\rangle \delta_+ u = (\gamma - 1) a U_c \langle \rho_r e^{-\epsilon/T}\rangle (\delta x/\langle u + a\rangle) \tag{10}$$

In Eq. (10), the angular brackets indicate that mean values should be taken for the increment. It is desirable to write the mean value of a quantity in terms of its initial value, indicated by an unbracketed symbol, plus one-half of its change, indicated by $d(\quad)/2$. Rewriting Eq. (10) and introducing the following approximation,

$$e^{(-\epsilon/T)(1 + dT/2T)^{-1}} \approx e^{-\epsilon/T}(1 + \epsilon dT/2T^2) \tag{11}$$

yields

$$\delta_+ p + (a + da/2)(\rho + d\rho/2)\, \delta_+ u = (\gamma - 1)\, \alpha U_c \times (\rho_r + d\rho_r/2) e^{-\epsilon/T} (1 + \epsilon dT/2T^2)(\delta x/\langle u + a\rangle) \tag{12}$$

If it is assumed that the propagation path for weak shocks is almost identical to the plus characteristic curve, then

$$\langle u + a\rangle \approx \langle u_s + M a_s\rangle = u_s + du_s/2 + (M + dM/2)(a_s + da_s/2) \tag{13}$$

where the subscript s indicates the steady-state flow properties in the gas before the passage of the shock. Substituting (13) into (11) and neglecting terms of higher order yields

$$\delta_+ p + a\, \rho\, \delta_+ u = \frac{(\gamma - 1)\, \alpha U_c\, \delta x}{M(a_s + da_s/2) + u_s} \times \rho_r e^{-\epsilon/T}\left(1 + \frac{\epsilon dT}{2T^2} + \frac{d\rho_r}{2\rho_r}\right) \tag{14}$$

Furthermore, if it is assumed that the relaxation time (the time required for the chemical process to pass from one equilibrium state to another) is much longer than the time required for the passage of the shock through the combustion region, then the chemical process may be retained as a function of the steady-state (undisturbed flow) properties. Thus Eq. (14) becomes

$$\delta_+ p + a\, \rho\, \delta_+ u = \frac{(\gamma - 1)\, \alpha U_c\, \delta x}{M(a_s + da_s/2) + u_s} \rho_{rs} e^{-\epsilon/T_s} \times \left[1 + \left(\frac{\epsilon}{2T_s}\right)\left(\frac{dT_s}{T_s}\right) + \frac{d\rho_{rs}}{2\rho_{rs}}\right] \tag{15}$$

The Rankine-Hugoniot relations and their total derivatives are presented below:

$$p = \rho_s a_s^2[2M^2/(\gamma + 1) - (\gamma - 1)/\gamma(\gamma + 1)] \tag{16}$$

$$\rho = (\gamma + 1)\rho_s M^2/[(\gamma - 1) M^2 + 2] \tag{17}$$

$$u = u_s + 2a_s (M - 1/M)/(\gamma + 1) \tag{18}$$

$$a = a_s K/(\gamma + 1) M \tag{19}$$

$$dp = 4\rho_s a_s^2 M\, dM/(\gamma + 1) + 2\rho_s a_s[2M^2/(\gamma + 1) - (\gamma - 1)/\gamma(\gamma + 1)]da_s + a_s^2[2M^2/(\gamma + 1) - (\gamma - 1)/\gamma(\gamma + 1)]d\rho_s \tag{20}$$

$$du = du_s + 2a_s(1 + 1/M^2)dM/(\gamma + 1) + 2(M - 1/M)da_s/(\gamma + 1) \tag{21}$$

Equations (18) and (21) assume that the positive flow direction is the direction of shock propagation.

Substituting Eqs. (17, 19, 20, and 21) into Eq. (15) yields

$$\frac{4}{\gamma + 1}\rho_s a_s^2 M dM + 2\rho_s a_s\left[\frac{2M^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma(\gamma + 1)}\right] da_s + a_s^2\left[\frac{2M^2}{\gamma + 1} - \frac{\gamma - 1}{\gamma(\gamma + 1)}\right] d\rho_s + a_s \rho_s \frac{(\gamma + 1) M^2}{(\gamma - 1) M^2 + 2} \times \left[du_s + a_s \frac{2}{\gamma + 1}\left(1 + \frac{1}{M^2}\right) dM + \frac{2}{\gamma - 1} \times \left(M - \frac{1}{M}\right) d a_s\right] = \frac{(\gamma - 1)\, \alpha U_c\, \delta x}{M(a_s + da_s/2) + u_s} \rho_{rs} e^{-\epsilon/T_s} \times \left[1 + \left(\frac{\epsilon}{2T_s}\right)\left(\frac{dT_s}{T_s}\right) + \frac{d\rho_{rs}}{2\rho_{rs}}\right] \tag{22}$$

Introducing the concentration of the reactant Y, defining the nondimensional function H associated with the rate of heat release by

$$H \equiv \frac{(\gamma^2 - 1)\, \alpha U_c Y_s e^{-\epsilon/T_s}}{M a_s^2 (a_s + da_s/2)} \delta x \tag{23}$$

and assuming that du_s/a_s and u_s/a_s are higher-order quantities, Eq. (21) becomes

$$\left[4M^2 + \frac{2K(M^2 + 1)}{(\gamma - 1)M^2 + 2}\right]\frac{dM}{M} = -\left[2M^2 + \frac{1}{\gamma} - 1 + \frac{K(M^2 - 1)}{(\gamma - 1) M^2 + 2} - \frac{H\epsilon}{2T_s}\right]\frac{dT_s}{T_s} + \left[\frac{H}{2} - 2M^2 - \frac{1}{\gamma} + 1\right]\frac{d\rho_s}{\rho_s} + \frac{H}{2}\frac{dY_s}{Y_s} + H \tag{24}$$

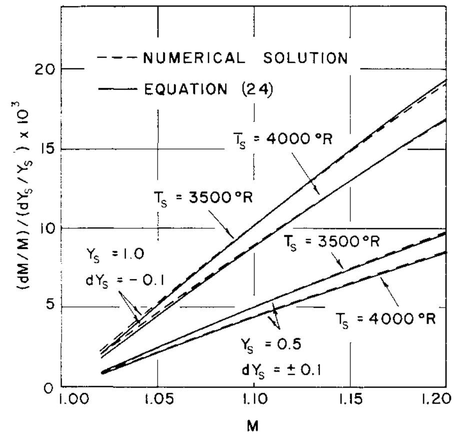

Fig. 1 Comparison of Eq. (24) with corresponding numerical solutions for the condition p_s = 500 psia, R = 67.7 ft-lb/lbm-°R, γ = 1.24, ϵ = 45,320 °R, $a = 5 \times 10^{10}$ sec^{-1}, U_c/c_p = 1500°R, $\dot{w}$ = 2.0 lb/sec-ft², $U_c dY = c_p dT$, and $d\rho_s/\rho_s = -dT_s/T_s$.

Results

Equation (24) gives the reduced change in shock Mach number dM/M as a function of dT_s/T_s, dY_s/Y_s, $d\rho_s/\rho_s$, shock Mach number M, and H that is always positive in an exothermic chemical reaction. When a weak shock-fronted pulse passes through a combustion region in one direction, reflects from a boundary, and passes back through the combustion region in the opposite direction, the change in dM/M produced by the temperature, density, and concentration gradients in the two directions may compensate (because the gradient will reverse signs), but the heat released by the chemical reaction will tend to amplify the pulse. The result is a net amplifying effect on a shock-fronted pulse propagating back and forth in the combustion chamber.

Figure 1 presents $(dM/M)/(dY_s/Y_s)$ vs M with different constant values of the parameters dY_s, Y_s, and T_s for the purpose of comparing Eq. (24) with the numerical solution. The numerical solution was obtained by the simultaneous numerical integration of Eqs. (6–9), employing an IBM 7094 computer; Eqs. (16–19), in conjunction with the steady-state temperature profile, were employed as a boundary condition, and it was assumed that the relaxation time for the chemical process was long compared to the time required for the passage of the shock. The conditions under which the two solutions were compared are listed under the figure. The two solutions are in satisfactory agreement; the difference diminishes from ~10% at M = 1.02 to <2% at M = 1.2. The curves are nearly straight lines which indicates that, for a given set of parameters, $dM \propto M^2$.

For the particular example chosen the value of H was small. Therefore, for the case of $Y_s = 0.5$, the difference between the curves for plus and minus dY is not apparent in Fig. 1. The numerical values from which the curves in Fig. 1 were plotted did show, however, that the curves for plus dY lie slightly above the corresponding curves for minus dY.

Equation (24) can also be employed to treat the less complex problem of a normal shock propagating through a gas having a gradient in temperature, but without chemical reaction. In that case, H is zero; and if the further restriction is made that $d\rho_s/\rho_s \approx -dT_s/T_s$, Eq. (24) reduces to the following relation:

$$\frac{dM/M}{dT_s/T_s} = \frac{-(\frac{1}{2})\,K(M^2 - 1)}{2(\gamma M^4 + 1) + K(M^2 + 1) - 2(M^2 - 1)^2} \qquad (25)$$

A plot of the right-hand side of Eq. (25) vs M is very nearly a straight line up through M = 1.2, indicating that, for a given dT_s/T_s and γ, $dM \propto M^2$. Equation (25) is rather insensitive to the specific-heat ratio over the range of values normally encountered; changing γ from 1.24 to 1.4 alters the results by <0.2%.

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Electric Propulsion Possibilities Using Steam Space Power

Henry R. Kroeger* and J. Frank Coneybear†
Astra, Inc., Raleigh, N. C.

Studies by Toms et al.[1] and others suggest that space power for electric propulsion may cost from 1 to 2 billion dollars. Considering costs and possible scheduling problems, it would seem that power for electric propulsion may be available late, if ever, if the present approach is continued, and that other approaches should be considered. This note presents a preliminary examination of systems consisting of electric thrusters and advanced steam space powerplants. The results indicate an advantageous matching of intrinsic characteristics.

Three power ranges are of interest for electric propulsion. Two of these are suitable for applications; one is suitable only for tests. Toms et al.[1] conclude that "the power requirements for electric rockets in the 1980's can be met by (a) a nuclear system in the 300–600-kw range having an initial 10,000 hour life and developing later to 14,000 hours with a shutdown capability for coasting periods, and (b) a larger system in the 2–6 Mw range an initial 10,000 hour life."

The necessity for proof-of-principle flight tests has been postulated, and it has been estimated that initial meaningful tests could be carried out with powers in the 10–30-kwe range.[2] Now, however, the close similarity of results obtained from ground tests and from the recent, successful space electric rocket tests (SERT) would seem to indicate a decreased need for further low-power tests.[3] Electric power in the range between 300 kwe and 6 Mwe can only be supplied by nuclear systems.[4] Powers in the 10–30-kwe range are low enough to be supplied by solar dynamic systems.[5,6]

Combined System Performance Characteristics

Lifetimes for steam-cycle systems will be longer than other approaches, because, as noted by the Atomic Energy Commission (AEC),[7] lower temperatures result in greater reliability, and because steam systems have higher over-all efficiencies. Figure 1 (derived from the Fig. 1 in Ref. 7) shows that pertinent steam components essentially can achieve the necessary 10,000- and 15,000-hr lifetimes, now. Note that liquid metal components must displace the temperature vs lifetime curve, in lifetime, by four orders of magnitude. The use of steam cancels the need for auxiliary cooling loops for the powerplant, the thruster, and the power con-

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* Vice-President.
† President. Member AIAA.